Sharpening Occam's Razor

by

Alan G. Herron, Ph. D.

A_Herron@telus.net

2005m02d28

Abstract

This paper examines the epistemological value of applying Occam's Razor, "essences should not be multiplied unnecessarily". The conclusion is that the principle can be stated with more precision, and extended, but does express an epistemological practice that is both sound and wise, and defensible on objective grounds.

Introduction

Occam's Razor states that "essences should not be multiplied unnecessarily" [1]. It is generally regarded as an intuitive maxim [2], not formally deducible from the laws of logic. But it should not be regarded as arbitrary, that is, without theoretical justification.

The purpose of this paper is to illustrate the nature of the errors and hazards blocked by use of Occam's Razor, and to supplement the idea to form more accurate and complete requirements.

Degrees of Freedom

To begin it is necessary to discuss a concept called "degrees of freedom".

Loosely, any descriptive model of a real system is considered to have one or more descriptive variables; premises or inputs that may be chosen arbitrarily, or are set by particular circumstances of a general case. One of the problems in establishing the logical structure of the descriptive model is to determine what these variables are, and how many of them should be incorporated in the theoretical model.

The general idea is that a real system being modeled is capable of being in a fixed number of states. The description, or theoretical model, connects the variables to these states. Ideally, specifying particular values for the variables will uniquely describe which state the real system is in. And changing the value of a variable will cause the system to change states.

Consider, for example, a glass, which may contain a number of marbles ranging from 0 to 99. Then the glass/marble system has 100 different possible states. The descriptive variable must be capable of distinguishing 100 different possibilities to model the real system accurately.

Suppose, instead, that there are 50 blue marbles, and 49 red, and we wish to model the colour information too. To describe the system we now need more than one variable. We need one to count the blue marbles, and one to count the red. The number of states now far exceeds 100. It is, in fact, 51 * 50 = 2550 (for each of the 51 possible choices for number of blues one has 50 choices for reds, giving 2550 in all).

In the single-colour case the number of degrees of freedom is one, with a range of values 0 through 99. In the two-colour system the number of degrees of freedom is 2. The first variable ranges over values 0 through 50. The second ranges over values 0 through 49.

This is a numeric example, but the same idea applies to logical systems, database tree models, mechanical or electrical networks, political or legal options, etc. In each case we have a conceptual model whose structure is a set of "free" inputs that, through relational constraints, imply a combination of outputs, or states of the system.

In general the number of degrees of freedom is the number of "free" variables required, i.e. necessary, or essential, to completely describe the system, without exceeding the system's variability. The variables can be counters, as in the examples above, premises in logical arguments, symbols representing measurements, etc. The variables must be capable of being chosen freely or set arbitrarily for a particular case. Taken together they contain all the information necessary to describe the state of the system.

Occam's "essences" are these degrees of freedom.

Modes of Failure in Developing a Model

If the exact nature of the real system is not known the number of variables is unknown. For a hypothetical model there are, then, three possibilities:

The number of variables in the model

1) is lower than the number in the real system
2) exactly matches the number in the real system
3) is higher than the number in the real system

Case 1: fewer variables in the model than in the real system.

The real system is under-specified by the model. As a result unexpected states may arise in controlled experiments or arguments. There would appear to be "noise", or unexpected variations, in our experimental results. If the discrepancy is severe we might even conclude our model is inappropriate when it is actually correct for the variables that have been correctly identified.

In short, the model will be partially predictive, but not deterministic.

Case 2: an exact match, the desired state of affairs.

The model is capable of describing every state of the real system, but only by using all the possible model variables. We would have perfect (deterministic) predictability and control, and no unexpected behaviour would arise if we use the model to manage the real system.

Case 3: too many variables in the model

This is the case where Occam's Razor comes into play as a control measure.

Experimentally, we would attempt to explore more model variables than we can generate in the real system, and find repeated states in the real system when our model predicts there should be no repetition. There will be no behaviour in the real system that compels us to add more variables if the model actually does include the correct subset of variables in its total suite.

As an aside: the repetition provides the information necessary to eliminate redundant variables. Whatever variable changes when the system does not is, at least, a candidate for elimination. The elimination is an application of Occam's Razor after the preliminary model has been built, whereas the usual application is in building the model in the first place.

In case 3, conceptually, we would have a model that is too complicated, hence more difficult or expensive to explore, and to understand. The system will appear unresponsive to some of our control efforts (when we change some of the redundant variables without altering the correct ones).

We also might conclude we have predictive capacity we do not, because we can choose values for our extra variables to match any observation we get.

An example of this, latter, failure mode is the science students' standard joke known as Cook's constant. This is a variable defined as "the published value divided by yours", and is used as a multiplier on experimental results. The effect, of course, is to "correct" any result to the published one. Cook's constant is a new degree of freedom, added to those of the legitimate analysis, with obviously devastating effect in regard to objectivity. (The name comes from the analogy to cooking, where "Cook" makes whatever dish (value) he/she wishes). Cook's constant is a joke, but it does illustrate the hazard of doing something similar in a serious argument*.

Conclusion

Case 3 demonstrates that Occam's razor is a sound principle of competent reasoning, and not just an intuitive whim. The number of independent variables cannot be quantified until we have the correct model, but the idea that we should not introduce extra variables unless and until we have good reason to do so is sound. If the restraint is not applied we can introduce, and perpetuate, errors that can be serious and damaging to our ultimate understanding.

A companion maxim covering Case 1 problems might be "don't reduce the number of variables below sufficiency". Together they amount to "get it right!"

In the interests of clarity I have simplified the description of modeling criteria somewhat. One must not only get the number of variables right, but the ranges of their values. Occam's Razor and the supplementary maxim should also be applied in the assignment of ranges of values.

The combination of correct choices for both variables and their ranges will result in an exact match of the number of possible states of the real system, giving the model fully deterministic predictive capability. In the light of the foregoing, Occam's Razor is a valuable, and powerful, epistemological principle for realizing this goal.

AGH

[1] *Quadlibeta Septem*, William of Occam, c. 1320. Translated by John Ponce of Cork in the 17th century. The original words are "*Essentia non sunt multiplicanda praeter necessitatem*"

[2] See, for example, "An Introduction to Logic" by H. W. B. Joseph, Second Edition, Revised. Oxford at the Clarendon Press reprinted 1970 p506 *et seq*.

* The concept of an omnipotent, omniscient, compassionate God may be seen as the Cook's constant of metaphysics. It appears to produce the desired results, always, but only at the expense of destroying all objectivity and respect for the Law of Identity. ☺